6.RP

EduTron Corporation

Draft for NYSED NTI Use Only

CHALLENGING PROBLEMS AND TASKS

6.RP RATIOS AND PROPORTIONAL RELATIONSHIPS

Understand ratio concepts and use ratio reasoning to solve problems

DRAFT 2012.11.29

Table of Contents

I. Challenging Problems from EduTron

Use with Care!

Before you dive into the first 12 problems, please be fully aware that these exemplars are provided as templates and models for you to **develop your own capacity to produce more** problems that are of the same caliber.

Dissect these problems carefully with your peers to build up your knowledge and skills so you can create similar "problems *with teeth*." The process of developing problems with rigor and depth is serious fun. Most of all, it will further improve your craft and refine your taste.

It is important to know your students well so you can create, find, or modify problems that best suit their needs. **Do not just rush to unleash these problems on your students! First, make sure YOU are really taking full advantage of them!**

Thank you.

Read me upside down, too.

The EduTron Team

I. Challenging Problems from EduTron

These problems could be used to make the Common Core Rigor and Mathematical Practice Standards come alive!

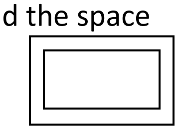
Fluency Concept Application MP1 MP2 MP3 MP4 MP5 MP6 MP7 MP8

1. Do you agree with the following? Why?

   a. The ratio of two people's weight remains the same, whether measured in pounds or kilograms.
   b. The ratio of two people's ages remains the same, whether this year, next year, or last year.
   c. The ratio of boiling temperature and freezing temperature of alcohol remains the same, whether measured in Fahrenheit or Centigrade.
   d. The ratio of gas consumption of two different cars remains the same, whether measured in miles per gallon or kilometers per liter.

2. Compare these fractions: $\frac{3}{4}$ and $\frac{3+1}{4+1}$. Why is one bigger than the other?

3. Are the two rectangles on the right similar? Assume constant width around the space between the two rectangles.

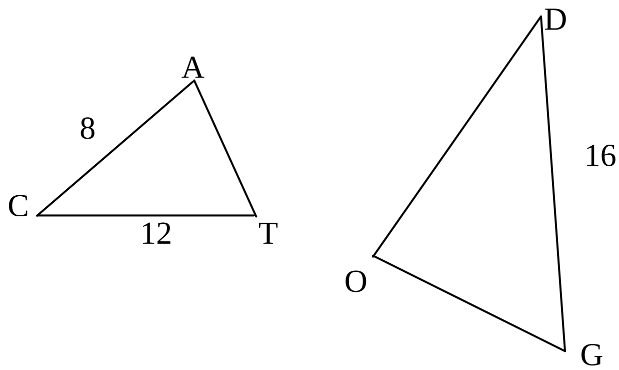


4. Two kindergarteners participated in a fund raiser event "*Walk Backwards for Hunger*" last weekend. John walked 2/3 of a mile and Mary walked 3/4 of a mile. Together, they raised $93.50. Please find out how much money John and Mary each raised. To be fair, the amount raised is proportional to distance walked.

5. What is the number that is exactly <u>2/5 of the way</u> going from 1/9 to 1/8 on the number line?
   Hint: If you go all the way, the number will be 1/8.

6. What is the point that is exactly <u>2/5 of the way</u> going from (-2, 3) to (2, -3) on the line containing both points?
   Hint: If you go all the way, the point will be (2,-3).

7. Use the triangles below to answer the question that follows.



If the two triangles ΔCAT and ΔDOG are similar, what is the measure of $\overline{DO}$?

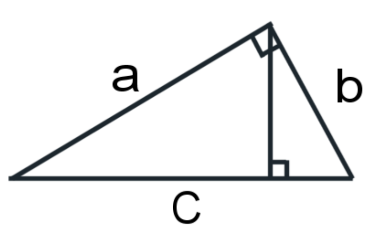
8. The ratio of Jim's money to Peter's money was 4:7 at first. After Jim spent 1/2 of his money and Peter spent \$60, Peter had twice as much money as Jim. How much money did Jim have at first?

9. The ratio of Jim's money to Peter's money was 4:7 at first. After Peter gave 3/14 of his money to Jim, they have equal amounts of money. How much money did Jim have at first?

10. The ratio of Jill's money and Kelly's money was 4:5. After they each spent \$156, the ratio became 8:11. How much money did Jill and Kelly have, respectively, at first?

11. How many grams of hydrogen are in 117 grams of water? (atomic weights: H: 1, O: 16)

12. In the following right triangle with side lengths a, b, and c, its height is shown.
    (a) How many similar triangles can you find in the following picture?
    (b) Prove Pythagoras Theorem using the results of part (a).
    (c) Find the lengths of the 3 unknown segments (i.e., the "altitude" and the 2 segments on side c) in terms of a, b, and c.



II. Classroom Tasks from Illustrative Mathematics

1. 6.RP Games at Recess

Alignment 1: 6.RP.A.1

The students in Mr. Hill's class played games at recess.

6 boys played soccer
4 girls played soccer
2 boys jumped rope
8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said,
"Four more girls jumped rope than played soccer."
Chaska said,
"For every girl that played soccer, two girls jumped rope."
Mr. Hill said, "Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio."

1. **Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.**

2. **Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.**

3. **Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.**

Commentary:

In a classroom where answers to problems in context are expected as complete sentences, and numerical values from a context are written with appropriate units, the task may not need to explicitly model and request it as these questions do.

While students need to be able to write sentences describing ratio relationships, they also need to see and use the appropriate symbolic notation for ratios. If this is used as a teaching problem, the teacher could ask for the sentences as shown, and then segue into teaching the notation. It is a good idea to ask students to write it both ways (as shown in the solution) at some point as well.

Solution:

1. Four more boys played soccer than jumped rope.

2. For every three boys that played soccer, one boy jumped rope. Therefore the ratio of the number of boys that played soccer to the number of boys that jumped rope is 3:1 (or "three to one").

3. For every two girls that played soccer, three boys played soccer. Therefore the ratio of the number of girls that played soccer to the number of boys that played soccer is 2:3 (or "two to three").

2. 6.RP Mangos for Sale

Alignment 1: 6.RP.A.2

A store was selling 8 mangos for $10 at the farmers market.

Keisha said,
“That means we can write the ratio 10 : 8, or $1.25 per mango.”
Luis said,
“I thought we had to write the ratio the other way, 8 : 10, or 0.8 mangos per dollar."
Can we write different ratios for this situation? Explain why or why not.

Commentary:

The purpose of this task is to generate a classroom discussion about ratios and unit rates in context. Sometimes students think that when a problem involves ratios in a context, whatever quantity is written first should be the first quantity in the ratio a:b. However, because the context itself does not dictate the order, it is important to recognize that a given situation may be represented by more than one ratio. An example of this is any problem involving unit conversions; sometimes one wants 3 feet : 1 yard and the associated unit rate 3 feet per yard and sometimes one wants 1 yard : 3 feet and the associated unit rate 1/3 yard per foot.

A similar task that provides students an opportunity to choose between the two different ratios and associated unit rates based on their usefulness is in development.

Solution:

Setting the record straight

Yes, this context can be modeled by both of these ratios and their associated unit rates. The context itself doesn’t determine the order of the quantities in the ratio; we choose the order depending on what we want to know.

3. 6.RP Price per pound and pounds per dollar

Alignment 1: 6.RP.A.2

The grocery store sells beans in bulk. The grocer's sign above the beans says, *"5 pounds for $4."*
At this store, you can buy any number of pounds of beans at this same rate, and all prices include tax.
Alberto said,
"The ratio of the number of dollars to the number of pounds is 4:5. That's $0.80 per pound."
Beth said,
"The sign says the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."

a. **Are Alberto and Beth both correct? Explain.**
b. **Claude needs two pounds of beans to make soup. Show Claude how much money he will need.**
c. **Dora has $10 and wants to stock up on beans. Show Dora how many pounds of beans she can buy.**
d. **Do you prefer to answer parts (b) and (c) using Alberto's rate of $0.80 per pound, using Beth's rate of 1.25 pounds per dollar, or using another strategy? Explain.**

Commentary:

This task could be used by teachers to help students develop the concept of unit rates. Its purpose is to help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero), and to encourage students to flexibly choose either unit rate, depending on the question at hand.

Item (d) admits many different answers and is intended to prompt a teacher-facilitated discussion of different student strategies, and so should likely be removed or reworded for assessment purposes, depending on the type of assessment involved. A productive discussion could develop around side-by-side comparisons of strategies that apply Alberto's rate and strategies that apply Beth's rate.

Solution:

Using a ratio table:

(a) Alberto and Beth are both correct. Their rates could be illustrated with a double number line or a ratio table like the following:

Pounds	Dollars
1	.80
1.25	1
2.5	2
5	4

(b) Double the quantities in Alberto's rate to find the price of two pounds:

Pounds	Dollars
1	.80
2	1.60

(c) Starting from Beth's rate and multiplying both quantities by ten shows the number of pounds that can be purchased for 10 dollars:

Pounds	Dollars
1.25	1
12.50	10

(d) Answers may vary. We can efficiently answer part (b) using Alberto's rate, and part (c) using Beth's rate.

4. 6.RP Friends Meeting on Bicycles

Alignment 1: 6.RP.A.3
Alignment 2: 6.RP.A.3.b

Taylor and Anya live 63 miles apart. Sometimes on a Saturday, they ride their bikes toward each other's houses and meet somewhere in between. Taylor is a very consistent rider - she finds that her speed is always very close to 12.5 miles per hour. Anya rides more slowly than Taylor, but she is working out and so she is becoming a faster rider as the weeks go by.

1. **On a Saturday in July, the two friends set out on their bikes at 8 am. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?**
2. **Make a table showing how far apart the two friends are after zero hours, one hour, two hours, and three hours.**
3. **At what time will the two friends meet?**
4. **Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? Is she correct?**
5. **A couple of months later, on a Saturday in September, the two friends set out again on their bikes at 8 am. Taylor, as always, rides at 12.5 miles per hour. This time they meet at 11 am. How fast was Anya riding this time?**

Commentary:

For sixth grade, this is presented as a series of problems leading up to the last one. This last question is appropriate without scaffolding for 7th grade; see "7.RP.3 Friends Meeting on Bikes."

Most students should be able to answer the first two questions without too much difficulty. The decimal numbers may cause some students trouble, but if they make a drawing of the road that the girls are riding on, and their positions at the different times, it may help.

The third question has a bit of a challenge in that students won't land on the exact meeting time by making a table with distance values every hour.

The fourth question addresses a useful concept for problems involving objects moving at different speeds which may be new to sixth grade students. This question provides one way to answer the next one, but not the only way. This question also addresses Standard for Mathematical Practice 3.

The story context is intended to make the problem more interesting to students, but it can also serve several mathematical purposes. A student who doesn't know where to start on a problem like this can guess or estimate using the story as a guide, and students who have found an answer can check it to see if it makes sense in the story. For example, Anya's speed in the second ride should be greater than in the first. Also, in comparing the two bike rides in July and in September, students can recognize that Taylor's speed doesn't change but Anya's does, and this changes the meeting time.

Solution:

Problem a, first method.
They started 63 miles apart, so 63−5.5−12.5=45 miles.

Solution:

Problem a, second method.
In the first hour, Anya went 5.5 miles and Taylor went 12.5 miles, so the two of them together went 18 miles, and 63−18=45 miles.

Solution:

Problem b, first method.
The table should show 63 miles at 0 hours, 45 miles at 1 hour, 27 miles at 2 hours, and 9 miles at 3 hours. Some of the students will probably recognize that the distance is decreasing by 18 miles each time, and this fact can be brought up in class discussion.

Solution:

Problem b, second method.
Here are a couple of ways to offer extra scaffolding to students who might need it.

Draw a line representing the route that Taylor and Anya will take, with their starting points labeled as being 63 miles apart. Then mark Taylor's position at 0, 1, 2 and 3 hours, and likewise for Tanya's.

Make a table with four columns instead of just two. The first column would be time in hours, then how far Anya has traveled, then how far Taylor has traveled, and finally the distance between them.

Solution:

Problem c, first method.
First, students should recognize from the table that the friends are 9 miles apart after three hours, and therefore they will meet in less than an hour. With a little more thought and discussion, they should recognize that 9 is half of 18, and so they will meet in 3.5 hours, or at 11:30 am.

If this 3.5 hour solution isn't clear, then one approach would be to re-do the table from Problem 2, but this time use half-hour increments instead of hour increments.

Solution:

Problem c, second method.
Since the friends are moving toward each other at 18 miles per hour, we can write 63/18=3.5 hours, so they will meet at 11:30.

Solution:

Problem d

What Taylor really means is that the distance between them is decreasing by 18 miles every hour, so the amount of time it will take them to meet is the same as if one person stays put and the other rides at 18 miles per hour. However, the place they meet will not be the same.

Solution:

Problem e

There are two shorter solutions listed for this problem in the 7th grade version. A student who doesn't see how to do the shorter versions might go back and make a table, as in Problem b. At 0 hours the friends are 63 miles apart, and at 3 hours they are 0 miles apart. Using this, a student could figure out the missing entries, and then reason that the friends are getting closer at 21 miles per hour. Since Taylor is riding 12.5 miles per hour, Anya must be riding 8.5 miles per hour. Adding the two extra columns described in the second solution of Problem b might also help.

5. 6.RP Running at a Constant Speed

Alignment 1: 6.RP.A.3
Alignment 2: 6.RP.A.3.b

A runner ran 20 miles in 150 minutes. If she runs at that speed,

1. **How long would it take her to run 6 miles?**
2. **How far could she run in 15 minutes?**
3. **How fast is she running in miles per hour?**
4. **What is her pace in minutes per mile?**

Solution:

Using a table

	A	B	C	D	E	F
Number of Minutes	150	15	7.5	30	45	60
Number of Miles	20	2	1	4	6	8

The values in column B were found by dividing both values in column A by 10. The values in column C were found by dividing both values in column B by 2. The other columns contain multiples of the values in column B.

1. If we look in column E, we can see that it would take her 45 minutes to run 6 miles.
2. If we look in column B, we can see that she could run 2 miles in 15 minutes.
3. If we look in column F, we can see that she is running 8 miles every 60 minutes (which is 1 hour), so she is running 8 miles per hour.
4. If we look in column C, we can see that her pace is 7.5 minutes per mile.

III. Selected 6.RP Problems for NYS

A. Oranges and Bananas

Aligned CCLS: 6.RP.3b, 6.RP.2
Item: MC

A grocery store sign indicates that bananas are 6 for $1.50, and a sign by the oranges indicates that they are 5 for $3.00. Find the total cost of buying 2 bananas and 2 oranges.

A $0.85
B $1.70
C $2.25
D $4.50

Key: B

Commentary:

This question aligns to CCLS 6.RP.3b and 6.RP.2 because students must find the unit price of each banana and each orange to determine the total cost of two of each item.

Rationale:

Option B is correct; two bananas cost $0.50 and two oranges cost $1.20.
Option A is the sum of the unit price of a banana and the unit price of an orange. Option C is half the sum of the given sale prices. Option D is the sum of the given sale prices.

B. Cutting Boards
Aligned CCLS: 6.RP.3d
Item: MC

Jeremy has two 7-foot-long boards. He needs to cut pieces that are 15 inches long from the boards. What is the greatest number of 15-inch pieces he can cut from the two boards?

A 15
B 10
C 11
D 12

Key: B

Commentary:

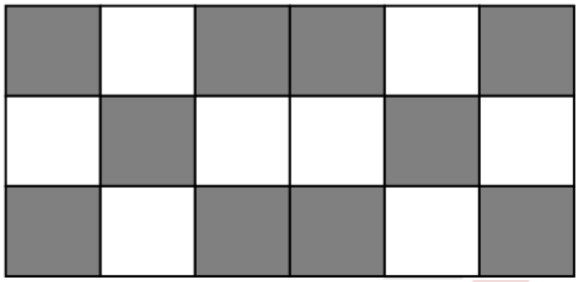
This question aligns to CCLS 6.RP.3d because it assesses a student's ability to use ratios for converting measurement units and to use reasoning skills and proportional thinking to make sense of the problem.

Rationale:

Option B is correct**.** Converting from feet to inches, the length of one of the boards is 7 × 12 = 84 inches. Thus, the largest number of 15-inch-long pieces that Jeremy can cut from one board is 5, because dividing 84 by 15 yields a quotient of 5 and a remainder of 9. It follows that the greatest number of pieces that Jeremy can cut from the two boards is 5 + 5 = 10. Option A is the number of sections from one board.
Options C and D represent miscalculations and/or not understanding the context.

C. Grey and White Tiles
Aligned CCLS: Part A and Part B: 6.RP.1; Part C: 6.RP.2
Item: CR

The new floor in the school cafeteria is going to be constructed of square tiles that are either gray or white and in the pattern that appears below:



Part A: What is the ratio of gray tiles to white tiles?
Answer: ____________________

Part B: What is the ratio of white tiles to the total number of tiles in the pattern?
Answer: ____________________

Part C: If the total cost of the white tiles is $12, what is the unit cost per white tile?
Answer: $____________________

Key:

Part A: 10 to 8, 5:4, or other equivalent ratio
Part B: 8 to 18, 4:9, or other equivalent ratio
Part C: $1.50 per white tile

Commentary:

This question aligns to CCLS 6.RP.1 and 6.RP.2 as it assesses a student's ability to apply the concept of ratio in a real-world situation. It requires that the student understand the concept and make sense of the situation.

Rationale:

Part A: The correct answer is a ratio of 10 gray tiles to 8 white tiles, or simplified, the ratio will be 5 gray tiles to 4 white tiles.
Part B: The correct answer is a ratio of 8 white tiles to 18 total tiles, or simplified, the ratio will be 4 white tiles to 9 tiles, in total.
Part C: Counting the tiles by color in the pattern above, it is found that there are 8 white tiles. If 8 white tiles cost $12, then the cost per white tile is $1.50.

IV. PARCC 6.RP Sample Item

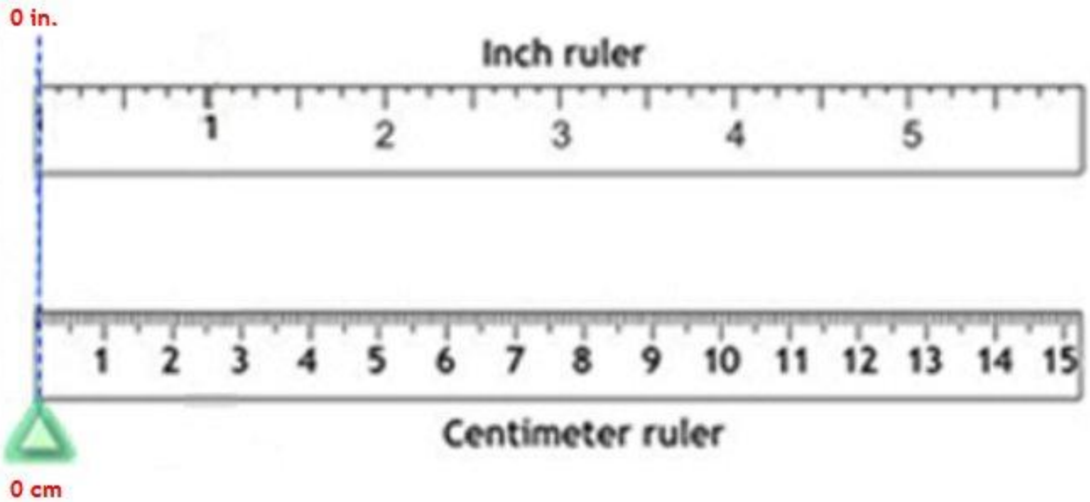
Type I: ***Tasks assessing concepts, skills and procedures***

http://www.parcconline.org/samples/mathematics/grade-6-mathematics

Grade 6 (Slider Ruler)

SAMPLE ITEM

Drag the slider to explore the relationship between the number of inches and the number of centimeters.



Select all of the statements that accurately represent the relationship between the number of inches and the number of centimeters.

- ☐ The ratio of centimeters to inches is 1 to 2.54.
- ☐ The ratio of centimeters to inches is 2.54 to 1.
- ☐ $i = 2.54c$, where i represents the number of inches and c represents the number of centimeters
- ☐ $c = 2.54i$, where i represents the number of inches and c represents the number of centimeters
- ☐ For every centimeter, there are 2.54 inches.
- ☐ For every inch, there are 2.54 centimeters.